FORMULA FOR CORRECTION OF A SMALL BIAS
in Q/I and U/I in weak field approximation

The effect of linear *cross-talk* of non-polarized intensity I onto the polarized signals for linearly polarized signals of Stockes Q and U (and also the circularly polarized Stockes V, which we ignore for the moiment) can be interpreted as in terms of their ratios to the overall intensity Q/I and U/I as addition of constants as

$$(Q/I) = (Q/I)^a + C_Q\,, \qquad (U/I) = (U/I)^a + C_U\,, \tag{1a,b}$$

where superscript ^{a} denotes the apparent signals, and C_Q and C_U some constants (in linear case). In *weak-field approximation*, these parameters Q/I and U/I are related to the magnitude of the transversal magnetic field B_T and azimuth angle ϕ as

$$Q \sim B_T^2 \cos 2\phi\,, \qquad U \sim B_T^2 \sin 2\phi\,, \tag{2a,b}$$

where a calibration constant is ignored. In case of small bias, their intrinsic values are getting some probably often but not always in general small errors δB_T and $\delta\phi$

$$B_T \to B_T + \delta B_T\,, \qquad \phi \to \phi + \delta\phi\,, \tag{3a,b}$$

which we would like to estimate equating the intrinsic transversal magnetic field B_T and azimuth angle ϕ with the apparent signals and using Eq. (2), into which we plug the anzats (1a,b) and (3a,b). Then ignoring second order perturbations with respect to these errors, after some linearization and algebra, we obtain the following expressions for the errors

$$\delta B_T = \frac{1}{2}\frac{C_Q(Q/I) + C_U(U/I)}{((Q/I)^2 + (U/I)^2)^{3/4}}\,, \qquad \delta\phi = \frac{1}{2}\frac{C_Q(U/I - C_U(Q/I))}{((Q/I)^2 + (U/I)^2)}\,. \tag{4a,b}$$

Now we can see that in most cases small bias creates small perturbations, except the cases when it causes a serious change in the azimuth angle. This causes a related problem of wrong or unstable $\pi-$disambiguation and further problems with magnetic field data interpretation.